8. Introduction to Trignometry - Board Exam Questions

The detailed solution for all the questions can be found at pschool.in/math/ncert/g10/chap-8

1 Mark Questions (MCQ)

1) Given that $\sin\theta = \frac{a}{b}$, then cos θ is equal to:
2) If $\cos A = \frac{1}{2}$, then the value of $\sin^2 A + 2\cos^2 A$ is:
3) If $\cos A = \frac{4}{5}$, then the value of tan A is:
4) If 2 sin A = 1, then the value of tan A + cot A is:
5) Given that $\sin 2\alpha = \frac{\sqrt{3}}{2}$, the value of sin 3α is:
6) The value of $\left(\frac{1}{2}\tan^2 45^\circ - \cos^2 60^\circ\right)$ is:
7) When $\sin A = \frac{1}{3}$, the value of cot A is
8) $\frac{1+\tan^2 A}{1+\cot^2 A}$ equals to:
9) For an acute angle θ, if $\sin\theta = \frac{1}{9}$, then value of $\frac{9\,\text{cosec}\,\theta + 1}{9\,\text{cosec}\,\theta - 1}$ is
10) Simplest form of $\frac{\sec A}{\sqrt{\sec^2 A - 1}}$ is
11) If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$, $0^\circ < A+B < 90^\circ$, $A > B$, then the value of A and B respectively are:
12) If θ is an acute angle and 7 + 4 sin θ = 9, then the value of θ is:
13) The value of $\tan^2\theta - \left(\frac{1}{\cos\theta} \times \sec\theta\right)$ is:
14) If $7\cos^2\theta + 3\sin^2\theta = 4$, then the value of θ is:
15) If $\sin 30^\circ \tan 45^\circ = \frac{\sec 60^\circ}{k}$, then the value of k is:
16) If $\tan 3\theta = \sqrt{3}$, then $\frac{\theta}{2}$ equals:
17) (cot θ + tan θ) equals:

18) tan 2A = 3 tan A is true, when the measure of ∠A is:

19) Which of the following statements is true?
(a) sin 20° > sin 70° (b) sin 20° > cos 20°
(c) cos 20° > cos 70° (d) tan 20° > tan 70°

20) $\frac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ}$ is equal to

21) If $x\left(\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}\right) = y\left(\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}\right)$, then x : y =

22) In a right triangle ABC, right-angled at A, if $\sin B = \frac{1}{4}$, then the value of sec B is

23) If $\sec\theta + \tan\theta = x$, then $\sec\theta - \tan\theta$ will be

24) If $2\sin 5x = \sqrt{3}$, $0^\circ \le x \le 90^\circ$, then x is equal to

25) If sec θ − tan θ = m, then the value of sec θ + tan θ is:

26) If cos (α + β) = 0, then value of $\cos\left(\frac{\alpha+\beta}{2}\right)$ is equal to:

27) If sin θ = cos θ, (0° < θ < 90°), then value of (sec θ · sin θ) is:

28) If $\cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\phi = \frac{1}{2}$, then tan (θ + φ) is:

29) If sin θ = 1, then the value of $\frac{1}{2}\sin\left(\frac{\theta}{2}\right)$ is:

30) If $\frac{x}{3} = 2\sin A$, $\frac{y}{3} = 2\cos A$, then the value of $x^2 + y^2$ is:

31) If $4\sec\theta - 5 = 0$, then the value of cot θ is:

32) If $\tan\theta = \frac{5}{2}$ then $\frac{4\sin\theta + \cos\theta}{4\sin\theta - \cos\theta}$ is equal to

33) If $\theta = 30^\circ$ then the value of $3\tan\theta$ is

34) If $2\tan A = 3$, then the value of $\frac{4\sin A + 3\cos A}{4\sin A - 3\cos A}$ is

35) $\left[\frac{3}{4}\tan^2 30^\circ - \sec^2 45^\circ + \sin^2 60^\circ\right]$ is equal to

36) $\sec\theta$ when expressed in terms of $\cot\theta$, is equal to:

37) If $\tan\theta = \frac{5}{12}$, then the value of $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$ is:

38) $\left(\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}\right)$ is equal to:

39) $\frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$, in simplified form, is:

40) If θ is an acute angle of a right angled triangle, then which of the following equation is NOT true?
(a) $\sin\theta\cot\theta = \cos\theta$ (b) $\cos\theta\tan\theta = \sin\theta$
(c) $\text{cosec}^2\theta - \cot^2\theta = 1$ (d) $\tan^2\theta - \sec^2\theta = 1$

41) $(\sec A + \tan A)(1 - \sin A)$ equals:

42) If $x\tan 60^\circ\cos 60^\circ = \sin 60^\circ\cot 60^\circ$, then $x =$

43) If $\sin\theta + \cos\theta = \sqrt{2}$, then $\tan\theta + \cot\theta =$

44) If $5\tan\beta = 4$, then $\frac{5\sin\beta - 2\cos\beta}{5\sin\beta + 2\cos\beta} =$

<u>Assertion and Reason</u>

1) ASSERTION (A): tan 2θ is not defined at θ = 45°.
REASON (R): sin 90° ≠ cos 90°.

2) ASSERTION (A): For an acute angle θ, $\sin\theta = \frac{3}{5} \Rightarrow \cos\theta = -\frac{4}{5}$.
REASON (R): For any value of θ, (0° ≤ θ ≤ 90°) $\sin^2\theta + \cos^2\theta = 1$.

3) ASSERTION (A): If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A = 1$.
REASON (R): $\sin^2 A + \cos^2 A = 1$

4) ASSERTION (A): If sin A = $\frac{1}{3}$ (0° < A < 90°), then the value of cos A is $\frac{2\sqrt{2}}{3}$.
REASON (R): For every angle θ, $\sin^2\theta + \cos^2\theta = 1$.

5) ASSERTION (A): For 0 < θ ≤ 90°, cosec θ − cot θ and cosec θ + cot θ are reciprocal of each other.
REASON (R): $\text{cosec}^2\theta - \cot^2\theta = 1$

2 Mark Questions

1) Evaluate: $\dfrac{3\cos^2 30^\circ - 6\,\mathrm{cosec}^2 30^\circ}{\tan^2 60^\circ}$

2) Evaluate: $\dfrac{\sin^3 60^\circ - \tan 30^\circ}{\cos^2 45^\circ}$

3) Evaluate: $\dfrac{\tan^2 60^\circ}{\sin^2 60^\circ + \cos^2 30^\circ}$

4) Evaluate: $2\sqrt{2}\cos 45^\circ \sin 30^\circ + 2\sqrt{3}\cos 30^\circ$

5) Evaluate: $2\sin^2 30^\circ \sec 60^\circ + \tan^2 60^\circ$.

6) Evaluate: $\dfrac{\cos 45^\circ + \sin 60^\circ}{\sec 30^\circ + \mathrm{cosec}\, 30^\circ}$.

7) Evaluate: $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \sin^2 60^\circ}$

8) Evaluate $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

9) Evaluate: $\dfrac{2\tan 30^\circ \cdot \sec 60^\circ \cdot \tan 45^\circ}{1 - \sin^2 60^\circ}$

10) Evaluate: $\dfrac{2\sin^2 60^\circ - \tan^2 30^\circ}{\sec^2 45^\circ}$

11) Evaluate the following: $\dfrac{3\sin 30^\circ - 4\sin^3 30^\circ}{2\sin^2 50^\circ + 2\cos^2 50^\circ}$

12) If $4k = \tan^2 60^\circ - 2\,\mathrm{cosec}^2 30^\circ - 2\tan^2 30^\circ$, then find the value of k.

13) If $4\cot^2 45^\circ - \sec^2 60^\circ + \sin^2 60^\circ + p = \dfrac{3}{4}$, then find the value of p.

14) Find the value of x if $2\,\mathrm{cosec}^2 30^\circ + x\sin^2 60^\circ - \dfrac{3}{4}\tan^2 30^\circ = 10$.

15) If $x\cos 60^\circ + y\cos 0^\circ + \sin 30^\circ - \cot 45^\circ = 5$, then find the value of $x + 2y$.

16) If $A = 60^\circ$ and $B = 30^\circ$, verify that:
$\sin(A + B) = \sin A \cos B + \cos A \sin B$

17) It is given that $\sin(A - B) = \sin A \cos B - \cos A \sin B$. Use it to find the value of $\sin 15^\circ$.

18) If $\tan\theta = \dfrac{24}{7}$, then find the value of $\sin\theta + \cos\theta$.

19) If $\sin A = y$, then express $\cos A$ and $\tan A$ in terms of y.

20) Use the identity: $\sin^2 A + \cos^2 A = 1$ to prove that $\tan^2 A + 1 = \sec^2 A$. Hence, find the value of tan A, when $\sec A = \frac{5}{3}$, where A is an acute angle.

21) If $\cot\theta = \frac{7}{8}$, then find the value of $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$.

22) If $\tan\theta = \frac{1}{\sqrt{5}}$, then find the value of $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$.

23) If $\tan A = \sqrt{3}$, where A is an acute angle, then find the value of $\frac{\sin^2 A}{1+\cos^2 A}$.

24) If $\sin\alpha = \frac{1}{\sqrt{2}}$ and $\cot\beta = \sqrt{3}$, then find the value of $\text{cosec}\,\alpha + \text{cosec}\,\beta$.

25) If $\sin(A+B) = 1$ and $\cos(A-B) = \frac{\sqrt{3}}{2}$, $0^\circ < A, B < 90^\circ$, find the measure of angles A and B.

26) If $2\sin(A+B) = \sqrt{3}$ and $\cos(A-B) = 1$, then find the measures of angles A and B. $0 \le A, B, (A+B) \le 90^\circ$.

27) If $\sin(A-B) = \frac{1}{2}$, $\cos(A+B) = \frac{1}{2}$; $0 < A+B \le 90^\circ$, $A > B$; find $\angle$A and $\angle$B.

28) If A and B are acute angles such that $\sin(A-B) = 0$ and $2\cos(A+B) - 1 = 0$, then find angles A and B.

29) If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$; $0^\circ < A+B < 90^\circ$; $A > B$, find A and B.

30) A, B, $A+2B$ and $2A+B$ are acute angles. If $\tan(A+2B) = \sqrt{3}$ and $\sin(2A+B) = \frac{1}{\sqrt{2}}$, find the measures of angles A and B.

31) Find an acute angle θ when $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$.

32) If $\tan\theta + \frac{1}{\tan\theta} = 2$, find the value of $\tan^2\theta + \frac{1}{\tan^2\theta}$.

33) If $\sin\theta + \cos\theta = \sqrt{3}$, then find the value of $\sin\theta \cdot \cos\theta$.

34) If $\cos A + \cos^2 A = 1$, then find the value of $\sin^2 A + \sin^4 A$.

35) Find the value of x for which
$(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = x + \tan^2 A + \cot^2 A$

36) If $a \sec\theta + b \tan\theta = m$ and $b \sec\theta + a \tan\theta = n$, prove that
$a^2 + n^2 = b^2 + m^2$

37) If $a \cos\theta + b \sin\theta = m$ and $a \sin\theta - b \cos\theta = n$, then prove that
$a^2 + b^2 = m^2 + n^2$.

38) Prove that: $\sqrt{\dfrac{1 - \sin\theta}{1 + \sin\theta}} = \sec\theta - \tan\theta$

39) Prove that: $\sqrt{\dfrac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

40) Prove that: $\sqrt{\dfrac{\sec A - 1}{\sec A + 1}} + \sqrt{\dfrac{\sec A + 1}{\sec A - 1}} = 2 \operatorname{cosec} A$

<u>3 Mark Questions</u>

1) Prove that: $\sqrt{\dfrac{\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + 1}} + \sqrt{\dfrac{\operatorname{cosec}\theta + 1}{\operatorname{cosec}\theta - 1}} = 2 \sec\theta$

2) Prove that: $\dfrac{\tan A}{1 + \sec A} - \dfrac{\tan A}{1 - \sec A} = 2 \operatorname{cosec} A$

3) Prove that $\dfrac{1 + \sec A}{\sec A} = \dfrac{\sin^2 A}{1 - \cos A}$.

4) Prove the following trigonometric identity:
$\dfrac{1 + \operatorname{cosec} A}{\operatorname{cosec} A} = \dfrac{\cos^2 A}{1 - \sin A}$

5) Prove that: $\left(\dfrac{1}{\cos A} - \cos A\right)\left(\dfrac{1}{\sin A} - \sin A\right) = \dfrac{1}{\tan A + \cot A}$

6) Prove that $\sec A(1 - \sin A)(\sec A + \tan A) = 1$.

7) Prove that: $\left(1 + \dfrac{1}{\tan^2\theta}\right)\left(1 + \dfrac{1}{\cot^2\theta}\right) = \dfrac{1}{\sin^2\theta - \sin^4\theta}$

8) Prove that: $\dfrac{\sin A + \cos A}{\sin A - \cos A} + \dfrac{\sin A - \cos A}{\sin A + \cos A} = \dfrac{2}{2\sin^2 A - 1}$

9) Prove that $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$.

10) Prove that: $\dfrac{\cos\theta - 2\cos^3\theta}{\sin\theta - 2\sin^3\theta} + \cot\theta = 0$

11) Prove that: $\frac{\sec^3\theta}{\sec^2\theta-1}+\frac{\operatorname{cosec}^3\theta}{\operatorname{cosec}^2\theta-1}=\sec\theta\cdot\operatorname{cosec}\theta(\sec\theta+\operatorname{cosec}\theta)$

12) Prove that $\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}=\sec\theta\operatorname{cosec}\theta-2\sin\theta\cos\theta.$

13) Prove that: $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\tan\theta+\cot\theta.$

14) Prove that: $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$

15) Prove that $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}=\frac{1-\sin\theta}{\cos\theta}.$

16) Prove that: $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\frac{1+\sin\theta}{\cos\theta}$

17) Prove that $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$

18) Prove that $\frac{\cos A+\sin A-1}{\cos A-\sin A+1}=\operatorname{cosec}A-\cot A$

19) Prove that $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec}A+\cot A.$

20) Prove that: $(\sin A+\sec A)^2+(\cos A+\operatorname{cosec}A)^2=(1+\sec A\operatorname{cosec}A)^2$

21) If $\sin\theta+\cos\theta=\sqrt{3}$, then prove that tan θ + cot $\theta=1$

22) If $\cos\theta+\sin\theta=1$, then prove that $\cos\theta-\sin\theta=\pm1.$

23) Given that sin θ + cos $\theta=x$, prove that
$\sin^4\theta+\cos^4\theta=\frac{2-(x^2-1)^2}{2}$

24) If $1+\sin^2\theta=3\sin\theta\cos\theta$, then prove that $\tan\theta=1$ or $\frac{1}{2}$.

25) If $\frac{1}{\sin x-\cos x}=\frac{\operatorname{cosec}x}{\sqrt{2}}$, then prove that
$\left(\frac{1}{\sin x+\cos x}\right)^2=\frac{\sec^2 x}{2}.$

26) If cot θ + cos $\theta=p$ and cot θ – cos $\theta=q$, prove that $p^2-q^2=4\sqrt{pq}$

27) If $\frac{\sec\alpha}{\operatorname{cosec}\beta}=p$ and $\frac{\tan\alpha}{\operatorname{cosec}\beta}=q$, then prove that $(p^2-q^2)\sec^2\alpha=p^2.$

28) If $x=h+a\cos\theta$, $y=k+b\sin\theta$, then prove that:
$\left(\frac{x-h}{a}\right)^2+\left(\frac{y-k}{b}\right)^2=1$

29) During a math class, Ms. Isha wrote the expression given below

on the board and asked the students to simplify it.

$$\frac{\cos\theta}{1-\sin\theta}+\frac{1-\sin\theta}{\cos\theta}$$

Jyoti solved it in her note book as follows:

$$= \frac{\cos^2\theta+(1-\sin\theta)^2}{(1-\sin\theta)\times\cos\theta} \quad \ldots(\text{step } 1)$$

$$= \frac{\cos^2\theta+\cos^2\theta}{(1-\sin\theta)\times\cos\theta} \quad \ldots(\text{step } 2)$$

$$= \frac{2\cos^2\theta}{(1-\sin\theta)\times\cos\theta} \quad \ldots(\text{step } 3)$$

$$= \frac{2\cos\theta}{1-\sin\theta} \quad \ldots(\text{step } 4)$$

Identify the step in which Jyoti has made error(s), if any. Rectify the same and find the correct answer.

30) Let $2A+B$ and $A+2B$ be acute angles such that $\sin(2A+B)=\frac{\sqrt{3}}{2}$ and $\tan(A+2B)=1$. Find the value of $\cot(4A-7B)$.